Radiation Effects on Vibrational Heating of Polymers

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The significance of the radiative heat transfer from a semitransparent material is assessed in the presence of nonlinear heat generation resulting from a cyclic loading. Critical limits for the heat generation parameter β are obtained for various values of the radiation-conduction parameters R and N. The effect of optically thin radiation is examined for a slab, while the optically thick limit is analyzed for the planar, cylindrical, and spherical geometries. An effective slip temperature coefficient is used for the combined conduction and optically thick radiation, and its importance on the critical limits for heat generation is assessed. It is concluded that the effect of radiative interchange is to increase significantly the critical values of β above those obtained from analysis involving only conduction.

Nomenclature

A_r = a constant related to the effect of temperature on viscosity
a_r = Rosseland absorption coefficient for optically thick limit
B = parameter in the expression for heat generation term
k = thermal conductivity of slab
k_p = radiation absorption coefficient in the optically thin limit
L = slab half thickness; L_c = characteristic length = L for planar geometry and R for cylindrical and spherical geometries
m = parameter in the expression for the heat generation term
n = material index refraction
N = radiation-conduction parameter for the optically thick limit, defined by Eq. (18)
q_c = conductive heat flux
q_r = radiative heat flux
$\dot{q}$ = rate of heat generation
R = radiative conductive parameter for optically thin limit, defined following Eq. (14)
T = temperature; T_s = surface temperature of slab; T_0 = temperature of slab at midplane ($x = L$); T_I = effective slip temperature at boundary
t = time
W = nonlinear heat generation term, defined by Eq. (1)
x = distance from the wall
β = dimensionless heat generation = $\gamma A_r L^2/k$; β_c = critical value of β
γ = $\frac{1}{2}\tau_0^2 \omega^{1-m} B$
δ = $A_r T_s$
ϵ_0 = strain amplitude
ξ = dimensionless distance = x/L for planar geometry and $= 1 - r/R$ for cylindrical and spherical geometries
σ = Stefan-Boltzmann constant
τ_0 = shear stress amplitude; τ_{xy}, τ_{rz} = shear stress in slab and cylinder, respectively
ϕ = dimensionless temperature = $A_r(T - T_s)$; $\phi_0 = A_r(T_0 - T_s)$; $\phi_I = A_r(T_I - T_s)$
ψ = dimensionless temperature = T/T_s; $\psi_0 = T_0/T_s$; $\psi_I = T_I/T_s$
ω = frequency of oscillating axial stress
Ω = defined by Eq. (42)

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Introduction

THE mechanical properties of materials are generally obtained by a well-established procedure of subjecting samples to a cyclic loading. It is important, however, that under the conditions of such tests, a thermally steady state is achieved in which the nonlinear rate of heat generation by the viscous resistance of the material is balanced by the rate of heat transfer from the material into the surrounding. If such a condition is not reached, the heat generated will exceed the heat transfer rate, leading to the disintegration of the sample in a manner similar to the heat explosion theory recognized in exothermic chemical reactions.[1-3]

For metals, a thermally steady state can be easily achieved in view of their high values of thermal diffusivities. However, for materials such as polymers, the behavior is quite different and such a steady state may not be attained under certain loading conditions.

The two established testing conditions are the constant strain amplitude method, ϵ_0 = constant, and the constant stress method, τ_0 = constant. It has been found that testing with a constant ϵ_0 does lead rapidly to a thermally steady state. However, testing with a constant τ_0 leads to certain critical states for the sample beyond which thermal disintegration (or explosion) takes place.

The heat transfer from an opaque solid to the ambient takes place chiefly by convection. Surface radiation plays an important role at increasingly higher sample temperature. When a solid is semitransparent, the heat transfer into the ambient takes place by both radiative volumetric emission from within the solid and by surface convection. Accordingly (although for an opaque solid subjected to a cyclic loading, a critical level exists for the heat generation rate above which thermal explosion of the solid takes place), a semitransparent solid with similar thermophysical properties exposed to the same loading conditions would be expected to experience a lower level of temperature. Hence, a higher level of loading can be imposed on such a solid before thermal disintegration takes place. In this paper, the significance of the radiative heat transfer from a semitransparent material is assessed in the presence of nonlinear heat generation resulting from a cyclic loading. Critical limits for the heat generation parameter β are obtained for various values of the radiation-conduction parameters R and N. The effects of optically thin radiation are examined for a slab of thickness of two L while the optically thick limit is analyzed for the planar, cylindrical, and spherical geometries in view of the similarities of formulation for the three geometries. It is important to emphasize that the present analysis reflects as well the trend in the values of the critical

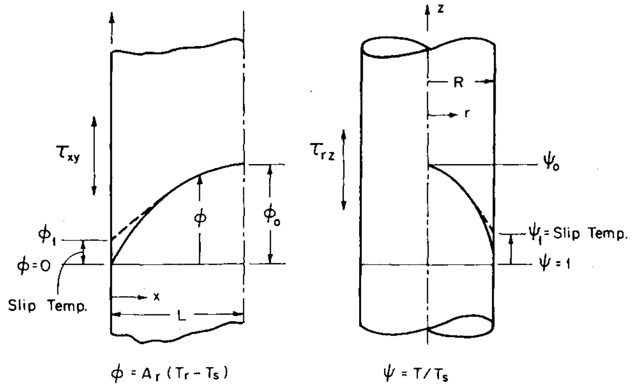

Fig. 1 Planar and cylindrical models chosen for the study.

limits for the heat generation parameter for other values of optical depths.

Analysis

Initially a flat sample of thickness of two L and exposed to a symmetric cyclic loading is chosen for the study as shown in Fig. 1. Constant thermophysical and optical properties are invoked. The length and width of the sample are taken to be much larger than its thickness, so that the heat transfer is essentially one-dimensional and symmetrical with respect to the midplane $x = L$. The cyclic heat generation in a polymetric material is generally represented by the following form[1,4]

$$W = \tfrac{1}{2}\tau_0^2 \omega^{1-m} B \exp[A_r(T - T_s)] \tag{1}$$

where ω is the frequency of the oscillating axial stress ($\omega = 2\pi\nu$) and τ_0 is the stress amplitude represented by

$$\tau_{xy} = \tau_0 \sin\omega t \tag{2}$$

If the shear stress is applied at time = 0 and if the inertia of the slab is neglected, the shear stress τ_{xy} is then independent of the coordinate x. The energy equation can then be written in the following form

$$\nabla \cdot (q_c + q_r) - \gamma e^{A_r(T - T_s)} = 0 \tag{3}$$

or

$$\frac{d^2T}{dx^2} - \frac{1}{k}\frac{dq_r}{dx} + \frac{\gamma}{k} e^{A_r(T-T_s)} = 0 \tag{4}$$

subject to the conditions: $x = 0$, $T = T_s$, $x = L$, and $dT/dx = 0$.

Multiplying Eq. (4) by dT/dx and integrating yields

$$\frac{1}{2}\left(\frac{dT}{dx}\right)^2 - \frac{1}{k}\int_{T_s}^{T}\frac{dq_r}{dx}\,dT + \frac{\gamma}{A_r k}\left(e^{A_r(T-T_s)} - 1\right) = C \tag{5}$$

At $x = L$, the temperature is τ_0 (still unknown) and $dT/dx = 0$ so that the constant C is given by

$$C = \frac{1}{k}\int_{T_s}^{T_0}\frac{dq_r}{dx}\,dT + \frac{\gamma}{A_r k}\left(e^{A_r(T_0 - T_s)} - 1\right) \tag{6}$$

and the expression for dT/dx becomes

$$\left(\frac{dT}{dx}\right) = +\left\{-\frac{2}{k}\int_{T_s}^{T_0}\frac{dq_r}{dx}\,dT + \frac{2\gamma}{A_r k}\left(e^{A_r(T_0-T_s)} - e^{A_r(T-T_s)}\right)\right\}^{1/2} \tag{7}$$

A plus sign is required on the right side of Eq. (7) because the temperature increases in the positive x direction. After the variables are separated, Eq. (7) becomes:

$$dT\bigg/\left\{-\frac{2}{k}\int_{T}^{T_0}\frac{dq_r}{dx}\,dT + \frac{2\gamma}{A_r k}\left(e^{A_r(T_0-T_s)} - e^{A_r(T-T_s)}\right)\right\}^{1/2} = dx \tag{8}$$

In the absence of radiation this equation does have a closed form solution.[4,5] If the following nondimensional variables are introduced

$$\phi = A_r(T - T_s); \qquad \xi = x/L, \qquad \beta = \gamma A_r L^2/k$$

along with $q_r = 0$, Eq. (8) becomes

$$d\phi/\{2\beta(e^{\phi_o} - e^{\phi})\}^{1/2} = d\xi \tag{9}$$

with the solution given by

$$e^{(\phi_o - \phi)/2} = \cosh\left[\left(\frac{\beta e^{\phi_o}}{2}\right)^{1/2}(1 - \xi)\right] \tag{10}$$

The temperature ϕ_0 at the centerline ($x = L$) is obtained from Eq. (10) by setting $\phi = 0$ when $\xi = 0$ to give

$$e^{\phi_o/2} = \cosh\left(\frac{\beta e^{\phi_o}}{2}\right)^{1/2} \tag{11}$$

This equation is plotted in Fig. 2 (curve labeled $R = 0$) for $\beta \le 0.88$. For $\beta > 0.88$, a steady-state solution does not exist, and hence thermal disintegration takes place.

Optically Thin Radiation

When radiation is considered in the optically thin limit, the expression for dq_r/dx becomes[6,7]

$$\frac{dq_r}{dx} = 4\sigma k_p n^2 (T^4 - T_s^4) \tag{12}$$

For constant absorption coefficient k_p, we can write

$$\int_{T}^{T_0}\frac{dq_r}{dx}\,dT = 4\sigma k_p n^2 \frac{T_0^5 - T^5}{5} - (T_s^4 T_0 - T_s^4 T) \tag{13}$$

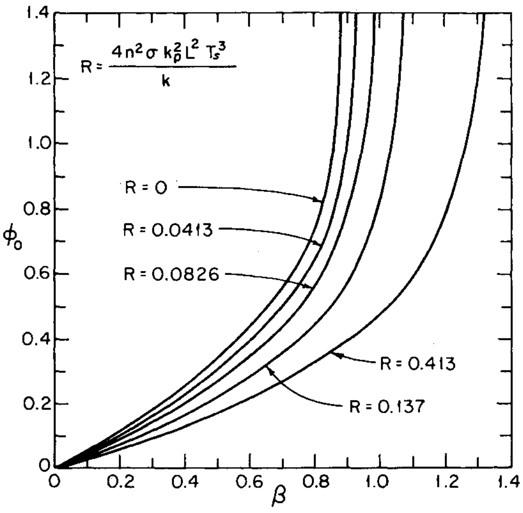

Fig. 2 Centerline temperature of slab ($x = L$).

and Eq. (8) becomes

$$\left\{-2\left(\frac{4n^2\sigma k_p}{k}\right)\left[\frac{T_0^5-T^5}{5}-T_s^4(T_0-T)\right]\right.$$
$$\left.+\frac{2\gamma}{A_r k}\left(e^{A_r(T_0-T_s)}-e^{A_r(T-T_s)}\right)\right\}^{-1/2}\mathrm{d}T=\mathrm{d}x \qquad (14)$$

With radiation present, it is more convenient to introduce the following variables

$$\psi=T/T_s,\qquad R=4n^2\sigma k_p L^2T_s^3/k;\qquad \xi=x/L;\qquad \delta=A_rT_s$$

so that Eq. (14) becomes

$$\mathrm{d}\psi\Bigg/\left\{-2R\frac{\psi_0^5-\psi^5}{5}-(\psi_0-\psi)+\frac{2\beta}{\delta^2}\left(e^{\delta(\psi_0-1)}-e^{\delta(\psi-1)}\right)\right\}^{1/2}$$
$$=\mathrm{d}\xi \qquad (15)$$

subject to the following conditions: $\xi=0$, $\psi=1$, $\xi=1$, and $\psi=\psi_0$.

Equation (15) was integrated numerically as discussed in the Results section. In presenting the results, however, for consistency with the no radiation case, the value of ϕ is plotted from the relation between ϕ and ψ as $\phi=\delta(\psi-1)$.

Optically Thick Radiation

The optically thick limit for radiative transfer is characterized by an optical depth $\gg 1$. Here the optical depth is defined as the product of a Rosseland absorption coefficient a_r and a characteristic length L_c, chosen as L for the planar geometry and as R for the cylindrical and spherical geometries. In this regime, the radiative interchange can be treated as a diffusion process with the radiative heat flux q_r given by Refs. 6 and 7.

$$q_r=-\left(16n^2\sigma T^3/3a_r\right)\nabla T \qquad (16)$$

Using this expression for q_r, the energy equation [Eq. (3)] in dimensionless form can be written as

$$\frac{1}{(1-\xi)^\alpha}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[(1-\xi^\alpha)(N\psi^3+1)\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right]+\frac{\beta}{\delta}e^{\delta(\psi-1)}=0 \qquad (17)$$

where

$N=16n^2\sigma T_s^3/3a_rk=$ radiation-conduction parameter for the optically thick limit (18)

$\alpha=0$ for planar geometry
$=1$ for cylindrical geometry
$=2$ for spherical geometry

The three geometries are treated together in this section in view of the similarity of formulation and the method of solution for the three cases. The appropriate boundary conditions for Eq. (17) are

$$\eta_l=\sigma T_s^4n^2\left[1-\psi_l^4\right]/q_{rl}\qquad \text{at } \xi=0 \qquad (19)$$

and

$$\frac{\mathrm{d}\psi}{\mathrm{d}\xi}=0\qquad \text{at } \xi=1 \qquad (20)$$

where η_l is an effective slip coefficient for combined conduction and optically thick radiation, ψ_l is the effective slip temperature at the wall and is to be determined, and q_{rl} is the radiative heat flux at the wall.

This form of boundary condition at $\xi=0$ is needed in view of the lack of validity of the optically thick limit at the boundary. Accordingly, an effective slip temperature results and must be accounted for when the diffusion limit is extended to the wall (see Fig. 1). Values for η_l are presented in Ref. 6 for any geometry as a function of the radiation-conduction parameter N. If the diffusion method is assumed to be applicable at the wall with no slip, the obvious boundary condition at the wall becomes: $\psi=1$ at $\xi=0$.

The expression for ψ_l is determined as follows. From the expression for the total heat flux q we have

$$q=q_r+q_c=-\left[\frac{16n^2\sigma T_s^3}{3a_r}+k\right]\frac{\mathrm{d}T}{\mathrm{d}x} \qquad (21)$$

and

$$q_{rl}=-\frac{16n^2\sigma T_s^3}{3a_r}\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{x=0} \qquad (22)$$

From Eq. (17), the expression for $\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{x=0}$ is found to be

$$\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{x=0}=\frac{T_s}{L_c}\frac{\beta}{\delta}\frac{1}{(N+1)}\int_0^1(1-\xi)^\alpha e^{\delta(\psi-1)}\mathrm{d}\xi \qquad (23)$$

Hence, q_{rl} becomes

$$q_{rl}=-\frac{N}{(N+1)}\gamma L_c\int_0^1(1-\xi)^\alpha e^{\delta(\psi-1)}\mathrm{d}\xi \qquad (24)$$

From Eq. (19), we can solve for ψ_l and obtain

$$\psi_l^4=1+\frac{16\eta_l}{3a_rL_c}\frac{\beta}{\delta}\frac{1}{(N+1)}\int_0^1(1-\xi)^\alpha e^{\delta(\psi-1)}\mathrm{d}\xi \qquad (25)$$

Equation (17) with the conditions given by Eqs. (19), (20), and (25) was integrated twice to give the following nonlinear integral equation

$$\psi=\psi_l-\frac{N}{4}\left(\psi^4-\psi_l^4\right)+\frac{\beta}{\delta}\int_\xi^1K_1(\xi,t)e^{\delta(\psi(t)-1)}\mathrm{d}t$$
$$+\frac{\beta}{\delta}\int_0^\xi K_2(\xi,t)e^{\delta(\psi(t)-1)}\mathrm{d}t \qquad (26)$$

where $K_1(\xi,t)$ and $K_2(\xi,t)$ are given by

planar: $K_1(\xi,t)=\xi \qquad K_2(\xi,t)=t$ (27)

cylindrical: $K_1(\xi,t)=-(1-t)\ln(1-\xi)$;
$K_2(\xi,t)=-(1-t)\ln(1-t)$ (28)

spherical: $K_1(\xi,t)=\dfrac{\xi(1-t)^2}{1-\xi}$;
$K_2(\xi,t)=t(1-t)$ (29)

The method of solution for Eq. (26) is discussed in the Results section of this paper.

Thermomechanical Behavior of a Cylindrical Medium with Distributed Masses

Of interest also is the case of a cylindrical medium exposed to a nonlinear heat generation of the form

$$\dot{q}=(1-\xi)^2e^{\delta(\psi-1)} \qquad (30)$$

This type of heat generation simulates the thermomechanical behavior of a cylinder with distributed masses. The energy equation for this problem becomes

$$\frac{1}{(1-\psi)}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[(1-\xi)(N\psi^3+1)\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right]+\frac{\beta}{\delta}(1-\xi)^2e^{\delta(\psi-1)}=0 \tag{31}$$

Integrating this equation twice yields

$$\psi=\psi_1-\frac{N}{4}(\psi^4-\psi_1^4)-\frac{\beta}{\delta}\int_0^{\xi}[\ln(1-t)]\times(1-t)^3e^{\delta[\psi(t)-1]}\mathrm{d}t-\frac{\beta}{\delta}\int_{\xi}^{1}[\ln(1-\zeta)]\times(1-t)^3e^{\delta[\psi(t)-1]}\mathrm{d}t \tag{32}$$

where ψ_1 is given by Eq. (25) with $\alpha=3$. In the absence of radiation, Eq. (31) has the following closed form solution[4,10] presented in terms of ϕ, $[\phi=\delta(\psi-1)=A_r(T-T_s)]$

$$\phi=-\ln\left\{\tfrac{1}{32}\beta\left[e^c+(1-\xi)^4e^{-c}\right]^2\right\} \tag{33}$$

with

$$c\equiv\left|\cosh^{-1}\left(\frac{8}{\beta}\right)^{1/2}\right| \tag{34}$$

At the center of the cylinder we set $\xi=1$ and obtain the expression for the maximum temperature ϕ_0 in the form

$$\phi_0=\ln\left(\frac{32}{\beta}\right)-2\ln\left[\left(\frac{8}{\beta}\right)^{1/2}+\left(\frac{8}{\beta}-1\right)^{1/2}\right] \tag{35}$$

Equation (35) shows that there exists a critical value for β, above which steady-state solutions do not occur. This critical value is $\beta_c=8$ at which $\mathrm{d}\phi/\mathrm{d}\beta=\infty$. This is shown in Fig. 4 for $N=0$ and $N=0.2$, the case labeled cylindrical II.

Approximate Solution for Small Values of N in Planar Geometry

Recalling Eq. (26) for planar geometry and evaluating that equation at the center of the slab where $\xi=1$ we get in terms of ϕ the following relation

$$\phi_0=\phi_1-\frac{N\delta}{4}\left[\left(\frac{\phi_0}{\delta}+1\right)^4-\left(\frac{\phi_1}{\delta}+1\right)^4\right]+\beta\int_0^1te^{\phi(t)}\mathrm{d}t \tag{36}$$

in the absence of radiation, $\phi_1=0$ and $N=0$, hence

$$\phi_0=\beta\int_0^1te^{\phi(t)}\mathrm{d}t \tag{37}$$

It was presented earlier by Eq. (11) that the expression for ϕ_0 in the absence of radiation can be written as

$$\phi_0=2\ln\left\{\cosh(\beta e^{\phi_0}/2)^{1/2}\right\} \tag{38}$$

In comparing Eqs. (37) and (38), it is found that with no radiation we have:

$$\beta\int_0^1te^{\phi(t)}\mathrm{d}t=2\ln\left\{\cosh(\beta e^{\phi_0}/2)^{1/2}\right\} \tag{39}$$

In the presence of radiation, the value of the integral on the left side of this equation will differ from the expression given by Eq. (39). If we assume that such a difference is small, we can then replace the integral in Eq. (36) by its equivalent from Eq. (39) and write

$$\phi_0\simeq2\ln\left\{\cosh\left(\frac{\beta e^{\phi_0}}{2}\right)^{1/2}\right\}-\frac{N\delta}{4}\left[\left(\frac{\phi_0}{\delta}+1\right)^4-\left(\frac{\phi_1}{\delta}+1\right)^4\right]+\phi_1 \tag{40}$$

Solving this equation for β yields

$$\beta=2e^{-\phi_0}\left\{\ln\left[e^{\Omega}+(e^{\Omega}-1)^{1/2}\right]\right\}^2 \tag{41}$$

where

$$\Omega=\left\{4(\phi_0-\phi_1)+N\delta\left[\left(\frac{\phi_0}{\delta}+1\right)^4-\left(\frac{\phi_1}{\delta}+1\right)^4\right]\right\}\Big/8 \tag{42}$$

With no radiation slip at the boundary, $\phi_1=0$, and Ω becomes

$$\Omega=\left\{4\phi_0+N\delta\left[\left(\frac{\phi_0}{\delta}+1\right)^4-1\right]\right\}\Big/8 \tag{43}$$

Results

Equation (15) is solved by iteration for a selected set of values for β and R. The results for the opaque solid are independent of A_r when plotted as ϕ vs β and ϕ vs ξ. However, for a semitransparent material, specification of δ or A_r is necessary, and, hence, a representative value for $A_r=0.02$ K^{-1} ($\delta=5.86$) was chosen and kept constant in the analysis. The solution begins by assuming an initial value for ψ_0 in the integrand of Eq. (15) and integration is then carried out using a fine-mesh Simpson's routine. R and β are taken as variables. The result of integration between $\psi=1$ and $\psi=\psi_0$ must be equal to 1 as the limits on ξ are zero and 1. If this condition is not met within a tolerance of 2%, a new value for ψ_0 is chosen, depending upon the results of integration as to whether it is greater or less than one. The integration is then repeated, with the cumulative integral (0 to ξ and 1 to ψ) being printed. When the imposed constraint is met, the value for ψ_0 is accepted as the correct one and the printed ψ vs ξ values are taken to represent the temperature distribution within the slab. The procedure is then repeated for new values of R and β. The singularity in Eq. (15) when $\psi=\psi_0$ can be removed using integration by parts. Most of the solutions did not require more than five iterations to converge to the correct value of ψ_0.

For the case of optically thick radiation, the nonlinear integral equation, Eq. (26), was solved by the method of successive approximations. Initially an approximate profile for $\psi(\xi)$ is assumed. ψ_1 is then calculated from Eq. (25) for specific values of N, β, δ, and the optical depth a_rL_c. Values for η_1 were obtained from Ref. 6 as a function of N. Equation (26) is then integrated for various values of ξ, where in this case the integration span 0 to 1 is divided into 100 divisions. A new set of values for $\psi(\xi)$ is generated and reused in Eq. (25) and subsequently in Eq. (26) until a tolerance of 2% on ψ is met. The results thus obtained are considered accurate enough for the present study.

Figure 2 shows the values of ϕ_0 with optically thin radiation as a function of β and for various values of the parameter R. When $R=0$, a critical value for β close to 0.88 is reached beyond which no steady-state solution exists. This value is translated ultimately into a level of frequency of the cyclic stress presented in Eq. (1). When R is greater than zero, the critical value for β shifts to a higher value, indicating the ability of the material to withstand a higher level of cyclic loading. This behavior is to be expected, since for a fixed value of β, radiation tends to decrease the temperature within the

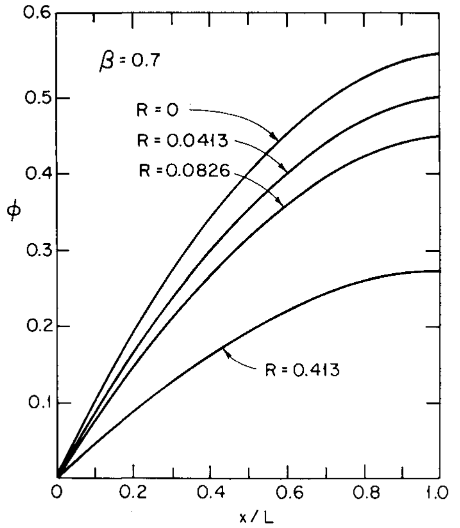

Fig. 3 Temperature distribution within the slab for various values of R.

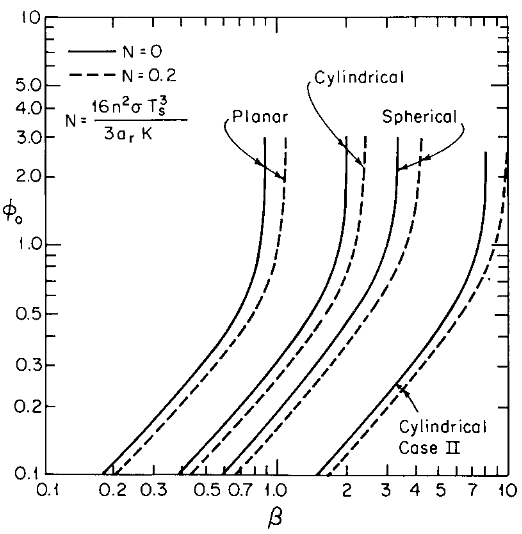

Fig. 4 Centerline temperature of the medium (optically thick radiation, $a_r L_c = 5$).

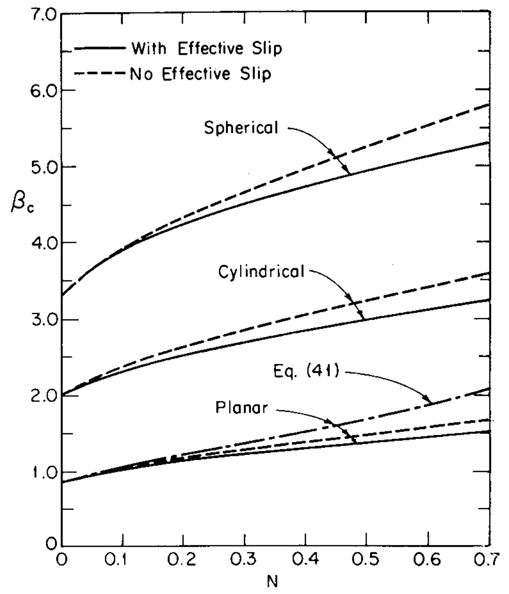

Fig. 5 Critical values of β_c as a function of N (optically thick radiation, $a_r L_c = 5$).

slab by providing an additional channel for heat liberation into the ambient.

Figure 3 shows the temperature distribution within the slab for various values of R and for fixed β and reflects the significance of radiation in reducing the level of the operating temperature. As conduction at the boundary is proportional to $d\phi/d\xi$ at $\xi = 0$, the slope of ϕ vs ξ continuously decreases at $\xi = 0$ with an increase in R indicating a reduction in the conduction heat flux while the total energy transfer is greater by virtue of the resulting lower value of ϕ. This behavior is consistent with previous work on coupled radiative conductive interaction problems (Refs. 8 and 9).

Figure 4 shows representative results for the planar, cylindrical, and spherical geometries when the medium is optically thick and for a radiation conduction parameter $N = 0.2$. Curves for other values of N showed similar trends, with β_c increasing with an increase in N. Although only planar and cylindrical geometries have direct applications in terms of nonlinear heat generation by cyclic loading, the three geometries considered and the analysis presented have direct applications in finding the critical conditions of ignition or thermal explosion criteria in reaction vessels undergoing exothermic chemical reactions. Consistent with the optically thin case presented earlier, the effect of radiation in the optically thick limit is also to increase the critical value of β beyond which no steady state exists and, hence, at which explosion takes place.

Figure 5 shows the critical values of β as a function of N within the range examined in the paper. It is seen that as N increases the critical value of β increases, but at a decreasing rate. Included in Fig. 5 are the results for β_c when it is invoked that the optically thick limit is applicable at the boundary with no effective slip. The results show that the no slip condition overestimates the critical limits for β by approximately 5 to 10% when the optical depth $a_r L_c$ is equal to five. It is important to indicate that the effective slip condition at the boundary decreases in significance when the optical depth progressively increases. This is seen in Eq. (25) in which $\psi_l \rightarrow 1$ as $a_r L_c \rightarrow \infty$. The approximate results represented by Eq. (41) with no slip for the planar geometry is also shown in Fig. 5. The curve shows that for values of N less than 0.4 the equation gives a reasonable accuracy within 10% of the exact results shown.

Conclusions

It is shown in the present study that for both optically thin and optically thick media, the effect of radiative interchange is to significantly increase the values of β_c above those obtained from analyses involving only conduction. Accordingly, for semitransparent materials with nonzero absorption coefficients undergoing vibrational heating, the radiative heat transfer process should be accounted for. The order of magnitude of the increase in β_c with increasing R or N is similar in both the optically thin and optically thick cases for the planar geometry. If the values of β_c for the optically thin limit are plotted vs R in Fig. 5, the results fall virtually on the same solid line generated for the optically thick case for values of R or N less than 0.3. This behavior indicates that the trend in the curves of β_c vs N or R is quite similar and can be expected to follow a similar trend for moderate optical depths.

A possible future extension of the present problem is to study the multidimensional thermal behavior of a short vibrating semitransparent element.

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